|  |
| --- |
| **Al-FARABI KAZAKH NATIONAL UNIVERSITY**Faculty of Mechanics and Mathematics**Department of Mathematical and Computer Modeling****SYLLABUS** Monte Carlo Methods and their Applications**Autumnal semester (First half-year) 2016 – 2017 academic year, the bachelor, 3 course**  |
| **Course code** | **Course name** | **Type** | Hour per week  | **Credits**  | **ECTS** |
| **Lecture**  | **Seminar** | **Laboratory**  |
|  | Monte Carlo Methods and their Applications | ED | **2** | **0** | **1** | **3** | **5** |
| Prerequisites | Mathematical Analysis, Algebra and Geometry, Information Science, Probability Theory and Mathematical Statistic, Stochastic Processes, ODE, PDE, Numerical Methods, Calculus, Calculations, Computations, Functional analysis.  |
| **Lecturer**  | **Kanat Shakenov, Doctor of Physical and Mathematical Science, Professor**  | **Office-time** | According to timetable |
| **e-mail:** | kanat.shakenov@gmail.com,shakenov.kanat@kaznu.kz  |
| **Phone**  | **+7 727 2211591, +7 705 182 3129** | **Lecture hall**  | **309** |
| **Teacher (laboratory studies)** | **Madina Abdykarim****+7 747 2876780** |  |  |
| **e-mail:** | **abdykarim@gmail.com** | **Lecture hall** | **304** |
| **Course description** | Monte Carlo statistical methods, particularly those based on Markov chains, have now matured to be part of the standard set of techniques used by statisticians. This course is intended to bring these techniques into the classroom, being a self-contained logical development of the subject, with all concepts being explained in detail, and all theorems, etc. having detailed proofs. There is also an abundance of examples and problems, relating the concept with statistical practice and enhancing primarily the application of simulation techniques to statistical problems of various difficulties.  |
| **Course aims** | That course give: random variable generation, Markov chains theory, definite integration, solutions: algebraic linearization system, second-type integral equation and boundary-value problem for elliptic equation. |
| **Learning outcomes**  | 1. Intimate knowledge of the algorithms of Monte Carlo Methods.
2. Ability simulate of the random variables along stochastic processes (along Markov Chains).
 |
| **References and resources**  | 1. Christian P. Robert, George Casella. Monte Carlo Statistical Methods. Second Edition. Springer. 2004.
2. K.K. Shakenov. Monte Carlo Methods and Applications. Methodical working. Almaty, KazSU, 1993. (In Russian).
3. I.M. Sobol’. Monte Carlo Method. Moscow, 1985. (In Russian).
4. I.M. Sobol’. Monte Carlo Numerical Methods. Moscow, Nauka, 1973. (In Russian).
5. S.M. Ermakov. Monte Carlo Methods and Adjacent Questions. Moscow, Nauka, 1975. (In Russian).
6. S.M. Ermakov, G.A. Mihailov. The Statistical Modelling. Moscow, Nauka, 1983. (In Russian).
7. B.C. Elepov, A.A. Kronberg, G.A. Mihailov, K.K. Sabelfeld. The solution of the boundary problem by Monte Carlo methods. Novosibirsk, Nauka, 1980. P. 171.
8. Sh. Smagulov, K.K. Shakenov. Monte Carlo Methods in Hydrodynamic and

Filtration Problems. Publishing House “Kazakh University”, 1999. P. 270. (In Russian).1. Feller W. An Introduction to Probability Theory and its Applications. Volume 1. John Wiley, New York. 1970. Volume 2. John Wiley, New York. 1971.
 |
| **Course organization**  | Structure of the course: 1.Lectures, 2. Laboratory**.** At a lectureto give the theoretical materials.At a laboratory to give stochastic calculations on PC. The homework may be preset (specified) according to the requirements. |
| **Course requirements**  | 1. The students at first of theoretical materials (lectures) attend. They must to know theoretical materials. 2. Next, to conduct PC Laboratory. Student with PC must construct the numerical model and graphic plot. 3. Student on one's own (or with teacher) must know how computational process analyses. To draw a right conclusion and the model identify.
 |
| **Grading policy**  | **Description of assignment** | **Weight**  | **Learning outcomes** |
| Individual tasksGroup projectAnalytical problem Examinations. Total | 35%10%15%40%100% | 1,2,34,5,62,3,44,5,61,2,3,4,5,6 |
| Your final score will be calculated by the formula $$The final grade on discipline=\frac{LC1+LC2}{2}∙0,6+0,1MT+0,3FC$$Below are minimum grades in percent:95% - 100%: А 90% - 94%: А-85% - 89%: В+ 80% - 84%: В 75% - 79%: В-70% - 74%: С+ 65% - 69%: С 60% - 64%: С-55% - 59%: D+ 50% - 54%: D- 0% -49%: F |
| **Discipline policy** | All work must be performed and defend within a specified time. Students who do not pass a regular job or received for his performance at least 50 % of points, have the opportunity to work on additional specified job schedule. Students who missed labs for a good reason, and spend their extra time in the presence of a laboratory, after the admission of the teacher. Students who have not complied with all types of work for the exam are not allowed. Also, take into account when assessing the activity and attendance of students during class Be tolerant and respect other people's opinions. Objections formulated in the correct form. Plagiarism and other forms of cheating are not allowed. Unacceptable prompting and copying during delivery SSS intermediate control and final exam, copying solved problems others, exam for another student. Student convicted of falsifying any information rate, unauthorized access to the Intranet using cribs, with a final grade «F».For advice on the implementation of independent work (SSS), and surrender their protection as well as for more information on the studied material and all other emerging issues by reading a course, contact the instructor during his office hours. |
| **Discipline schedule** |
| **Week**  | **Topic** | **Number of hours** |  **Maximum grade**  |
| **1 – 2**  | **Lecture 1 – 4.** ElementaryProbability Theory.Probability. Examples. Definition and illustrations. Deductions from the axioms. Independent events. Arithmetical density. Examples. Exercises. Random Variables. Examples. Definition of Random Variables. Distribution and Expectation. Definition of Mathematical Expectation. Examples. Integer-valued random variables. Examples. Random variables with densities. General case. Exercises.  | **4** | **8** |
| **1 – 2** | **Laboratory 1 – 4.** Distributions.Equiprobability distribution, Even/uniform distribution,Binomial, Poisson, Geometric, Cauchy, Conditional, Conjugate, Dirichlet, Discrete, Exponential, Generalized inverse normal, Generalized inverse Gaussian, Isotropic vector in 3D space. | **4** | **8** |
| **1 – 2** | **Students self-instruction (SSI) by subject (Homework, Project beginning etc. ) 1 – 4.**Any kind type of RVG. Computer simulation.  |  | **4** |
| **3 – 4** | **Lecture 5 – 8.** Methods of simulations of random variables. Pseudo-Random Number Generator. Uniform Random Variable on the interval . Uniform Simulation. Algorithm a Uniform Pseudo-Random Number Generation. The Inverse Transform. Optimal Algorithms. General Transformation Methods. Accept-Reject Methods. The Fundamental Theorem of Simulation. The Accept-Reject Algorithm. Problems. Random Walks. Markov Chains. Transition probabilities. Basic structure of Markov chains.  | **4** | **8** |
| **3 – 4** | **Laboratory 5 – 8.** Random Walks. Markov Chains. Computer simulation of Markov Chains.  | **4** | **8** |
| **3 – 4** | **SSI 5 – 8.** Computer simulation of Markov Chains. |  | **4** |
| **5 – 6** | **Lecture 9 – 12.** Introduction. Tchebyshev inequality. Law of . Classic Monte Carlo Integration. Importance Sampling. Estimated variance. Principles. Finite Variance Estimators. Comparing Importance Sampling with Accept-Reject. Laplace Approximations. Problems. Notes. | **4** | **8** |
| **5 – 6** | **Laboratory 9 – 12.** Estimated of Integral  by Monte Carlo methods.  | **4** | **8** |
| **5 – 6** | **SSI 9 – 12.** Computer simulation of Estimated of Integral. |  | **4** |
| **7 – 11**  | **Lecture 13 –22.** The Reduced LAES. Sequential Methods of the Solutions LAES. The Discrete Markov Chains. Stationary Chains. Chain Condition. Reversibility and the Detailed Balance Condition. Kac’s Theorem. Ergodicity and Convergence. Ergodic Theorems. Central Limit Theorems.  | **10** | **20** |
| **7 – 11**  | **Laboratory 13 –22.** Algorithm of solutions of LEAS by Monte Carlo methods.  | **10** | **20** |
| **7 – 11**  | **SSI 13 –14.** Computer simulation of Estimated of solutions of LEAS by Monte Carlo methods.  |  | **10** |
|  | **IC 1** |  | **100** |
|  | **Midterm Exam** |  | **100** |
| **12 – 15**  | **Lecture 23 –30.** The Continuous Markov Chains and the Second-Order Integral Equations (IE). The Algorithm of the Solution. The Biased Estimator of the Solution. The Theorem of the Variance. Estimations of the Solution of Integral Equation. Exercise of solutions IE by Monte Carlo methods. The Solution of the Boundary Value Problem of Poisson Equation and Helmholtz Equation by Monte Carlo Methods. Green function of Helmholtz operator of the ball. Algorithms “Random walks on spheres” and “Random walks on lattices”. Continuous Markov Chains. Algorithm of solution of Dirichlet problem for Poisson equation. Algorithm of solution of Dirichlet problem for Helmholtz equation. Theorem of Variance. Estimation of derivatives on the solutions by Monte Carlo methods.  | **8** | **16** |
| **12 – 15**  | **Laboratory 15 –16.** Algorithm of solutions of Integral Equations by Monte Carlo methods. Estimations of the Solution of Integral Equation. Exercise of solutions IE by Monte Carlo methods. Computer simulation of the algorithm “Random walks on spheres” and “Random walks on lattices”. Computer simulation of the solution of Dirichlet problem for Poisson equation. Computer simulation of the solution of Dirichlet problem for Helmholtz equation. Algorithm of estimations of derivatives on the solutions by Monte Carlo methods.  | **8** | **16** |
| **12 – 15**  | **SSI 15 –16.** Computer simulation of the algorithm “Random walks on spheres” and “Random walks on lattices”. Computer simulation of the solution of Dirichlet problem for Poisson equation. Computer simulation of the solution of Dirichlet problem for Helmholtz equation. Computer simulation of the algorithm of estimations of derivatives on the solutions by Monte Carlo methods. Computer simulation of the algorithm of solution of Dirichlet problem for Poisson equation by algorithm “Random walks on lattices”. |  | **8** |
|  | **IC 2** |  | **100** |
|  | **Exam**  |  | **100** |
|  | **Total** |  | **100** |

##### Reviewed at the department meeting

***Report №\_\_ from «\_\_» \_\_\_\_\_\_\_\_\_\_\_\_2016***

**Head of department D. Zhakebayev**

**Lecturer K. Shakenov**